B.Sc. Semester-IV

US04CSTA21

Unit - III

Time series Analysis

<u>Time Series:</u>

A set of data depending on time is called a time series

OR

Time series is an arrangement of Statistical data in a chronological order i.e. in accordance with occurrence of time.

Most of the series relating to economics, business and commerce are the examples of time series. e.g. the series related to prices, production and consumption of various commodities, agricultural and industrial production, national income and foreign exchange, profit of business houses, bank deposits, prices and dividend of shares, shares in stock markets etc. are all time series spread over a long period of time.

Mathematically a time series is defined by the functional relationship y = f (t) where y is the value of the phenomena or variable under consideration at a time t. e.g. (i) The population (y) of a country or a place in different year t (ii) No. of births or death (y) in different months t of a year (iii) the sale (y) of a departmental store in different months t of the year (iv) The temperature (y) of a place in different days t has etc. constitute the time series.

Thus if the value of a phenomena or variable at time $t_1, t_2 \ldots t_n$ are $Y_1, Y_2, \ldots, Y_n$ respectively then

t	t_1	t_2	………………	t_n
Y	y_1	y_2	………………	y_n

<u>Utility/Importance/Purpose of Time-series</u>

(a) **It helps in understanding the past behavior of time-series**
(b) **It helps in understanding the present situation**
(c) **It helps in predicting the future values of the series**
(d) **It is helpful for comparison**
(e) **The study of time-series is useful to government for planning and framing suitable policies for future.**

Que: What do you mean by Time Series analysis?

Time series analysis is a device through which an effort is made to isolate (separate out) the various components (factors or forces) which influence the time series variable in the series of data and measure, if possible.

For example, the price of any commodity (rice) depends upon many factors like demand, supply, rainfall, the price of substitute items, political changes etc. All these factors influence the price of rice in their own way, and hence the change in the price of rice is influenced by the totality of all these factors. If all these factors or components can be separate out from one another, then it may be possible to know the effect of each component on the price and they may help us in predicting the future values of the series.

<u>Components of Time series:</u>

The various components of time series are:

(i) Trend or Secular trend or long term variation (T)

(ii) Seasonal variation (S)

(iii) Cyclic variation (C)

(iv) Irregular or Random variation (I)

(i) Trend or Secular trend or long term variation (T)

Changes that have occurred as a result of general tendency of the data to increase or decrease, known as trend or secular trend.

(ii) Seasonal variation (S)

Changes that have taken place during a period of 12 months as a result of change in climate, weather conditions, festivals etc. Such changes are called seasonal variations.

(iii) Cyclic variation (C)

Changes that have taken place as a results of booms (to grow suddenly and rapidly) and depressions. Such changes are called cyclic variations.

(iv) Irregular or Random variation (I)

Changes that have taken place as a result of forces that could not be predicted like floods, earthquakes (natural calamities), famines (a severe shortage of food) etc. Such changes are called irregular or random variations.

Components of Time series:

The four components of time series are:

1. Secular trend

2. Seasonal variation

3. Cyclical variation

4. Irregular variation

Secular trend: A time series data may show upward trend or downward trend for a period of years and this may be due to factors like increase in population, change in technological progress, large scale shift in consumer's demands, etc. For example, population increases over a period of time, price increases over a period of years, production of goods on the capital market of the country increases over a period of years. These are the examples of upward trend. The sales of a commodity may decrease over a period of time because of better products coming to the market. This is an example of declining trend or downward trend. The increase or decrease in the movements of a time series is called Secular trend.

Seasonal variation: Seasonal variations are short-term fluctuation in a time series which occur periodically in a year. This continues to repeat year after year. The major factors that are responsible for the repetitive pattern of seasonal variations are weather conditions and customs of people. More woolen clothes are sold in winter than in the season of summer .Regardless of the trend we can observe that in each year more ice creams are sold in summer and very little in winter season. The sales in the departmental stores are more during festive seasons that in the normal days.

Cyclical variations: Cyclical variations are recurrent upward or downward movements in a time series but the period of cycle is greater than a year. Also these variations are not regular as seasonal variation. There are different types of cycles of varying in length and size. The ups and downs in business activities are the effects of cyclical variation. A business cycle showing these oscillatory movements has to pass through four phases-prosperity, recession, depression and recovery. In a business, these four phases are completed by passing one to another in this order.

Irregular variation: Irregular variations are fluctuations in time series that are short in duration, erratic in nature and follow no regularity in the occurrence pattern. These variations are also referred to as residual variations since by definition they represent what is left out in a time series after trend, cyclical and seasonal variations. Irregular fluctuations results due to the occurrence of unforeseen events like floods, earthquakes, wars, famines, etc.

Which components of a time series would you mainly associate each of the following? Why?

1. **A decline in ice-cream sales during November to February-----Seasonal**
2. **Fall in death rate due to advances in Science -----Trend**
3. **A strike in a factory delaying production for 10 days -----Irregular**
4. **A fire in a factory delaying production for three weeks -----irregular**
5. **Inflation -----Cyclic (Inflation means a rise in prices)**
6. **An increase in employment during harvest time ----Seasonal**
7. **Rainfall in Delhi that occurred for a week in Dec-1979 -----Irregular**
8. **A decrease in price during harvest time -----Seasonal**
9. **Recession -----Cyclic (Recession means a period during which trade and industrial activity in a country are reduced)**

Measurements of Trend

Following are the methods of measuring trend.

- **(i) Graphical or Free hand curve method**
- **(ii) Semi-average method**
- **(iii) Moving average method**
- **(iv) Least square method**

Mathematical models for time-series:

The following are the two models commonly used for the decomposition of a time-series into its components.

Additive Model

According to additive model the time-series can be express as

$$\boldsymbol{Yt = Tt + St + Ct + It}$$

Where Yt is the time-series value at time t and $Tt, St, Ct\ and\ It$ represent the trend, seasonal, cyclic and irregular (random) variation at time t respectively. The additive model assumes that all the four factor of time series operate independently of each other.

However this assumption is not true in most of economic and business time-series.

Multiplicative Model

To overcome the limitation of the additive model, multiplicative model is framed. Most of the economic and business time-series are characterize by the following classical multiplicative model

$$\boldsymbol{Yt = Tt \times St \times Ct \times It}$$

This model assumes that the four components of the time-series are due to different causes but they are not necessarily independent and that can affect each other. In this model $S, C\ and\ I$ are not viewed as absolute amount but rather a relative variation.

Methods (Measurements) of studying Trends:

The various methods of studying trends are:

- **(i) Graphical Method or Free hand curve method**
- **(ii) Method of Moving Average**
- **(iii) Method of Least Square**

(i) Graphical Method or Free hand curve method:

In this method the pairs of observations are plotted as a point on the graph paper by taking time (t) along x-axis and values of variable y along y-axis. Then the smooth free hand curve is drawn passing through maximum no. of points in such a way almost equal numbers of points are left on either side of the curve. The curve drawn in this way indicates the trend.

This method is very simple as it doesn't involve any mathematical formula or calculation but it is purely subjective i.e. different person may draw different curves for the same time-series data and because of this, it is not much applicable.

(ii) Semi average method:

(iii) Method of Moving Average:

<u>The method of moving average is use to eliminate the short term variation i.e. seasonal variation and cyclic variation</u>. Following are the steps for method of moving average.

Step-1: To obtain the moving average the group of beginning year say k (usually known as period of moving average) which constituted a business cycle is chosen for calculating average. This average is placed in the middle of k year.

Step-2: Now omit first year value from the group and include the succeeding year value in the group. Again calculate the average of the group and place it in front of the middle year of the respective group.

Step-3: if the number of years in a group (i.e. period of moving average (k) is odd, then there is no problem of locating the middle year, but if the number of year in the group is even then no single year is the middle year. Hence to overcome this average of the average as in pair is calculated and placed against the middle year of the two years. In this way we get the moving averages against years.

Step-4: Repeat the above procedure until all the observations are exhausted.

Step-5: These moving averages are considered as the trend values. Plotting all the moving average on the graph paper and joining the successive point we get the graph of trend values.

Advantages:

1. **The main advantage of this method is that it eliminates the short term variation.**
2. **It is easy to calculate and also adding some more observations in the time-series doesn't require the redo the entire calculation.**
3. **This method is not subjective.**

Disadvantages:

1. **The main disadvantage of this method is that we can't obtain trend values for all the given period (i.e. for the beginning and end of the data depending on the period of the moving average).**
2. **This method can't be use for prediction of future values.**
3. **In any time-series the cycles are not regular but they are taken to be regular.**

(iv) Method of Least Squares:

In this method we fit a suitable curve (straight line or second degree parabola) by least square techniques to the given time-series data considering time t as an independent variable and time-series variable Y as dependent variable.

Straight line: (Same as in Curve fitting)

Second degree parabola: (same as in curve fitting)

Remark:

The estimated value of Y obtain from the equation (straight line or second degree parabola) fitted to the given data for given value of time t represents the trend value and is denoted by $\hat{Y}$. Also the fitted equation is useful to estimate or predict the future value of the variable for the future time t.

For mathematical simplification use following transformation on the time period t.

$$Xi = \begin{cases} \dfrac{ti - \text{middle time period}}{C.I}, & \text{if } n \text{ is odd} \\ \dfrac{ti - \text{mean of two middle time period}}{\frac{1}{2}C.I}, & \text{if } n \text{ is even} \end{cases}$$

$i = 1, 2 \dots n$; **Where n: number of pairs of observations, $C.I$ interval between two successive values of t.**

Advantages

1. **Because of mathematical treatment this method completely eliminate the subjectively.**
2. **The trend by this method gives very accurate interpolated values.**
3. **The trend equation can be used to estimate or predict the value of the variable for any time period.**

Disadvantages

1. **This method requires more calculation.**
2. **If some more observations are added in the given data then to determine new trend value for the modified data you have to repeat the entire calculation.**

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* Measurements of Trend

Following are the methods of measuring trend.

(i) Graphical method or Free hand curve method
(ii) Semi-average method
(iii) Method of moving average
(iv) Method of Least square.

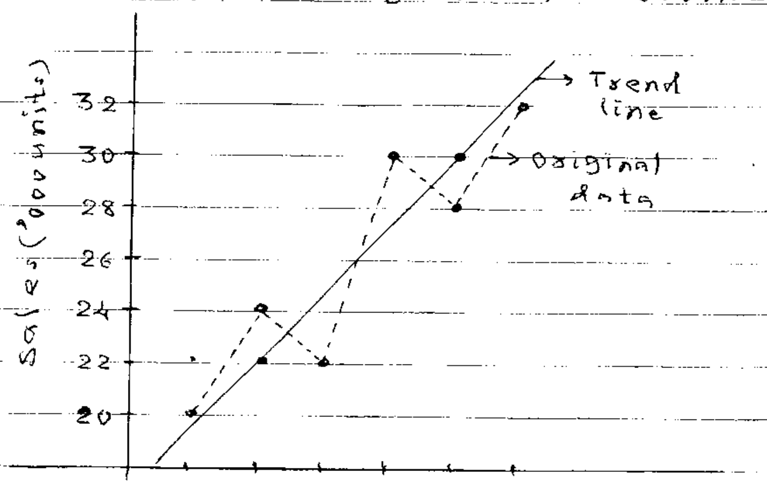
⇒ Examples:

(semi-average method)

1. Determine trend of the following data by using semi-average method and estimate the sale for the year 2007.

Year	Sales ('000 units)	Semi-average
2001	20	
2002	24	66/3 = 22
2003	22	
2004	30	
2005	28	90/3 = 30
2006	32	

Here the semi-average 22 is to be plotted against the mid-year of the first part i.e 2002 and the semi-average 30 is to be plotted against the mid year of second part i.e 2005.



In this method, the whole time series data is classified into two equal parts w.r.to time. However, in case of odd no. of years, the equal parts are obtained on

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Remark:

→ (First way)
The trend values for different years can be obtained from the trend line graph.

→ Alternate way:

The average increment in the value of sales ('000 units) for 3 years from 2002 to 2005 is $30 - 22 = 8$

$\therefore$ Average yearly increment in sales = $8/3 = 2.6667$

computation of trend values ('000 unit)

Year	Trend values ($\hat{y}$) ('000 units)
2001	$22 - 2.6667 = 19.3333$
2002	22
2003	24.6667
2004	27.3333
2005	30
2006	32.6667

$\therefore$ Trend value (estimated sales) for 2007

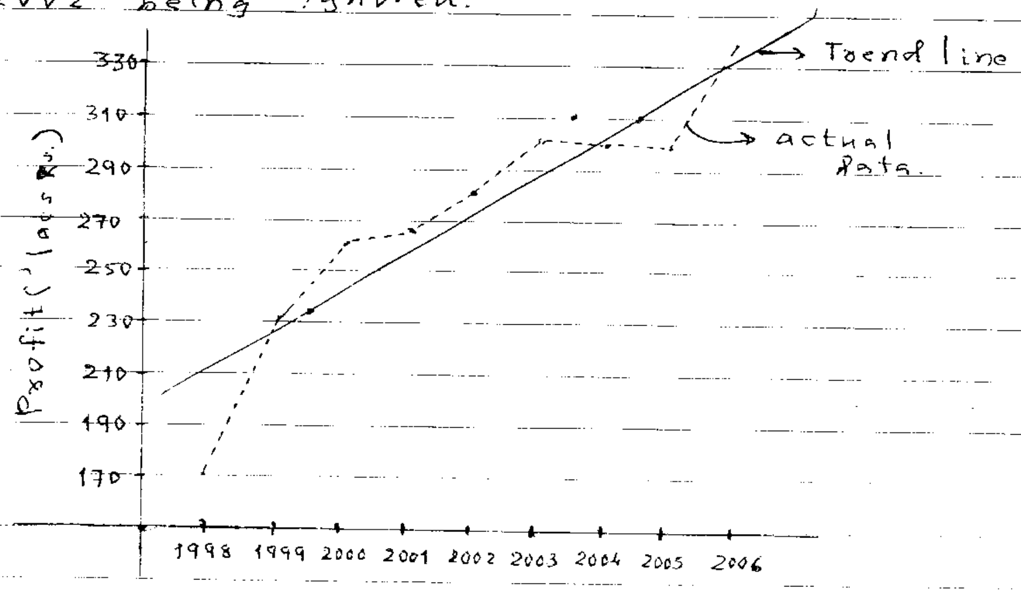
$= 32.6667 + 2.6667 = 35.3334$

≈ 35 ('000 units)

(2) From the following data find the trend values by the method of semi-averages. Also, estimate the profit for 2007.

Year (t)	profit (y) ('lacs Rs.)	semi-average	Trend values ($\hat{y}$) ('lacs Rs.)
1998	170		208.6
1999	231		224.2
		232.25 ≈ 232	232
2000	261		239.8
2001	267		255.4
2002	278		271
2003	302		286.6
2004	299		302.2
		309.75 ≈ 310	310
2005	298		317.8

Here the no. of year 9 is odd. The two middle parts will be 1998 to 2001 and 2003 to 2006, the value for middle year 2002 being ignored.



From the above table, we observe that the average increment in the value of profits ('lacs Rs.) for 5 years is 310 - 232 = 78

∴ Avg. yearly increment in profit = $\frac{78}{5}$ = 15.6 ('lacs Rs.)

∴ Trend value (Estimated profit) for the year 2007 = 349 ('lacs Rs.)

⇒ Trend using Moving average method.

(1) From the following data calculate the 3-yearly moving average and determine the trend values.

OR

Calculate trend value by 3-yearly moving average from the following data.

Year	1980	1981	1982	1983	1984	1985
Sales ('000 units)	5	7	9	12	11	10

Let t: Year
y: Sales ('000 units)

calculation of trend
(using moving average)

Year (1)	Sales (y) (2)	3-yearly moving totals (3)	3-yearly moving avg. (4)=(3)/3 (Trend value $\hat{y}$)
1980	5	–	
1981	7	21	7
1982	9	28	9.33
1983	12	32	10.66
1984	11	32	10.66
1985	10	29	9.66
1986	8	30	10.00
1987	12	33	11.00
1988	13	42	14.00
1989	17	49	16.33
1990	19	50	16.66
1991	14	–	–

(4) calculate the trend by moving average method, assuming 4-yearly cycle

OR

From the following data calculate 4-yearly moving average and determine the trend values.

Year	Sales ('lacs rs.)	Year	Sales ('lacs rs.)
1991	500	1996	540
1992	520	1997	560
1993	550	1998	570
1994	470	1999	590
1995	510	2000	610.

(P.T.O)

* For calculation see the printed (computerised) material available to you.

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sol^n:

Computation of 4-yearly moving average (m.a)

Year (1)	sales ('Lacs Rs.) (2)	4-yearly moving totals (3)	centered moving totals (4)	Trend values (centered m.a) (5) = (4) ÷
1991	500		—	
1992	520		—	—
		2040		
1993	550		4090	511.25
		2050		
1994	470		4120	515.00
		2070		
1995	510		4150	518.75
		2080		
1996	540		4260	532.50
		2180		
1997	560		4440	555.00
		2260		
1998	570		4590	573.75
		2330		
1999	590		—	—
2000	610		—	—

⇒ Method of Least square:

calculation of trend using method of least square.

see the material (Page - 87)

⇒ Seasonal variation by simple average method.

(1) compute, the seasonal, indices/ seasonal, variation (S.I) using simple average method, for the following time-series data.

(Monthly data)

Month	1994	1995	1996	Monthly avg. ($\bar{x}_i$)	Monthly S.I $= \bar{x}_i/\bar{x} \times 10$
Jan.	15	23	25	21	70
Feb.	16	22	25	21	70
Mar.	18	28	35	27	90
Apr.	18	27	36	27	90
May	23	31	36	30	100
June	23	28	30	27	90
July	20	22	30	24	80
Aug	28	28	34	30	100
Sept	29	32	38	33	110
Oct.	33	37	47	39	130

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K = no. of years = 3
n = no. of months = 12

Monthly avg. for i^{th} month

$$\bar{x}_i = \frac{1}{K}\sum_{i=1}^{K} x_i \quad , i = 1, 2, .. n = 1, 2, .. 12$$

Overall average (mean of all means)

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n}\bar{x}_i$$

Seasonal indices for i^{th} month

$$= \frac{\bar{x}_i}{\bar{X}} \times 100$$

e.g. seasonal index for January $= \frac{\bar{x}_1}{\bar{X}} \times 100.$

(2) Calculate the seasonal indices from the following data using the simple average meth

(Quarterly data)

K = no. of year = 5
n = no. of quar = 4.

Year	Q_1	Q_2	Q_3	Q_4
1994	72	68	80	70
1995	76	70	82	74
1996	74	66	84	80
1997	76	74	84	78
1998	78	74	86	82
Quarterly avg. $(\bar{x}_i)$	75.20	70.40	83.20	76.80
Quarterly S.I $\left(\frac{\bar{x}_i}{\bar{X}} \times 100\right)$	98.43	92.15	108.90	100.52

overall avg. $\bar{X} = 76.40$

Σ S.I $= 400$

H.W (3) The following are the number of books borrowed from a public library, on six working days, for a period of 5 weeks. compute the seasonal (daily) index for the series.

Week	No. of books borrowed (hundreds) Mon	Tue	Wed	Thu	Fri	Sat
1	2.5	4.3	4.4	4.6	5.1	6.2
2	1.8	3.4	5.2	4.9	5.3	6.8
3	1.2	2.5	4.8	5.1	6.2	7.0
4	1.9	2.2	4.9	6.1	7.1	6.9

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⇒ Calculation of seasonal indices using Ratio to trend method.

(1) Using 'Ratio to trend' method, determine the quarterly seasonal indices for the following data.

Production of coal ('in million tons)

Year	Q_1	Q_2	Q_3	Q_4
1993	30	40	36	34
1994	34	52	50	44
1995	40	58	54	48
1996	54	76	68	62
1997	80	92	86	82

Soln Let t: year

y: Avg. production of coal ('million tons)

(computation of yearly trend values)

Year (t)	Production of coal (y)	$u = t - 1995$	u^2	uy	Trend values ($\hat{y}$)
1993	35	-2	4	-70	32
1994	45	-1	1	-45	44
1995	50	0	0	0	56
1996	65	1	1	65	68
1997	85	2	4	170	80
Total	$\Sigma y_i = 280$	$\Sigma u_i = 0$	$\Sigma u_i^2 = 10$	$\Sigma u_i y_i = 120$	

Let the straight line (Linear) trend equation be

$$y = a + bt \quad \text{—— (I)}$$

where a and b are unknown constants.

Here n = no. of pairs = 5 (odd)

Let us make the transformation (t to u)

$$u_i = t_i - 1975 \quad , \quad n = 5 \text{ (odd)}$$

∴ The new modified equation of linear trend be

$$y = a' + b'u \quad \text{—— (II)}$$

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Using method of least squares, the normal equations for estimating a' and b' are

$$\Sigma y_i = n a' + b' \Sigma x_i$$
$$\Sigma x_i y_i = a' \Sigma x_i + b' \Sigma x_i^2$$
$$\therefore \hat{a}' = \cancel{56}\ \Sigma y_i / n = \frac{280}{5} = 56$$
$$\hat{b}' = \Sigma x_i y_i / \Sigma x_i^2 = 12$$

Hence the estimated trend equation be

$$\hat{y} = 56 + 12x \quad \text{——— (A)}$$

From the trend equation (A), we observe that avg. yearly increment in the trend values $= \hat{b}' = 12$ = yearly increse ~~in the productio~~ of coal

$\therefore$ avg. Quarterly increment in the trend $= \cancel{0}\ \hat{b}'/4 = 3$.

computation of quarterly trend values ($\hat{y}$)

Year	Q1	Q2	↓	Q3	Q4
1993	27.5	30.5	32	33.5	36.5
1994	39.5	42.5	44	45.5	48.5
1995	51.5	54.5	56	57.5	60.5
1996	63.5	66.5	68	69.5	72.5
1997	75.5	78.5	80	81.5	84.5

Ratio to trend (in %)
($y/\hat{y} \times 100$)

Year	Q1	Q2	Q3	Q4
1993	109.09	131.15	107.46	93.15
1994	86.08	122.35	109.89	90.72
1995	77.67	106.42	93.91	79.34
1996	85.04	114.29	97.84	85.52
1997	105.96	117.20	105.52	97.04
Quarterly avg. ($\bar{x}_i$)	92.77	118.28	102.92	89.15

overall avg. ($\bar{x}$) =

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H) Calculation of seasonal indices using "Ratio to moving avg." method

(1) Calculate seasonal indices using ratio to moving average method from the following data.

Production of cars ('000 units) (Quarterly data)

Year	Q1	Q2	Q3	Q4
2001	68	61	61	63
2002	65	58	66	61
2003	68	63	63	67

Computation of trend values (using moving average method)

Ratio to m.a. (6) ($y/\hat{y} \times 100$)	Year (t) (1)		Production of cars (2) (y)	4-Quarterly moving totals (3)	Centered moving totals (4)	(Trend values) 4 Quarterly moving averages ($\hat{y}$) (5) = (4) ÷ 8
—	2001	Q1	68	—		
—		Q2	61	—	—	—
97.02		Q3	61	253	503	62.87
101.41		Q4	63	250	497	62.12
104.21	2002	Q1	65	247	499	62.37
92.43		Q2	58	252	502	62.75
104.97		Q3	66	250	503	62.87
95.50		Q4	61	253	511	63.87
106.05	2003	Q1	68	258	513	64.12
97.67		Q2	63	255	516	64.50
—		Q3	63	261	—	—
—		Q4	67	—	—	—

Computation of seasonal indices (using ratio to moving average)

Year	Q1	Q2	Q3	Q4	
2001	—	—	97.02	101.41	
2002	104.21	92.43	104.97	95.50	
2003	106.05	97.67	—	—	
Quarterly average ($\bar{x}_i$)	105.13	95.05	100.99	98.45	Overall avg. ($\bar{x}$) = 99.90
Quarterly S.I ($\bar{x}_i/\bar{x} \times 100$)	105.23	95.14	101.09	98.54	Σ S.I = 400

Ex: The following are the number of books issued from a public library, on six working days, for a period of six weeks compute the seasonal daily index using ratio to trend method.

Week	No. of books issued ('hundreds)					
	Mon	Tue	Wed	Thu	Fri	Sat
1	2.5	4.3	4.4	4.6	5.1	6.2
2	1.8	3.4	5.2	4.9	5.3	6.8
3	1.2	2.5	4.8	5.1	6.2	7.0
4	1.9	3.2	4.9	6.1	7.1	6.9
5	2.1	3.2	4.3	5.3	6.1	7.2
6	1.2	3.1	2.7	4.8	5.3	6.2

Solution Let t : week

y : Avg. ~~weekly~~ no. of books issued per week ('hundreds)

: Avg. weekly no. of books issued ('hundreds)

Computation of weekly trend

Week (t)	y	$u = \frac{t-3.5}{0.5}$	u^2	uy	weekly trend values ($\hat{y}$)
1	4.5167	−5	25		4.6677
2	4.5667	−3	9		4.5995
3	4.4667	−1	1		4.5313
4	4.8500	1	1		4.4631
5	4.7000	3	9		4.3949
6	3.8833	5	25		4.3267
	$\Sigma y = 26.9834$	$\Sigma u = 0$	$\Sigma u^2 = 70$	$\Sigma uy = -2.3838$	

Let the equation of straight line (linear) trend be

$$y = a + bt \quad \text{——— (I)}$$

where a and b are unknown constant.

where t - mean of two middle terms

$$u = \frac{\qquad\qquad}{\frac{1}{2}\,(c.i)}, \quad n = 6 \text{ (even)}$$

$$= \frac{t - 3.5}{0.5}$$

$\therefore$ New modified equation of linear trend be

$$y = a' + b'u \quad \text{———} \quad (II)$$

where a' and b' are unknown constants.
Using the method of least squares, the normal equations for estimating a' and b' are:

$$\Sigma y_i = n a' + b' \Sigma u_i$$

$$\Sigma u_i y_i = a' \Sigma u_i + b' \Sigma u_i^2$$

$$\therefore \hat{a}' = \Sigma y_i / n = \frac{26.9834}{6} = 4.4972$$

$$\hat{b}' = \Sigma u_i y_i / \Sigma u_i^2 = -0.0341$$

Hence the estimate trend eq^n be = avg. weekly decrease

$$\hat{y} = 4.4972 - 0.0341\,u$$

Computation of daily trend values.

Week	Mon	Tue	Wed	½	Thu	Fri	Sat
1	4.6819	4.6762	4.6705	4.6677	4.6649	4.6592	4.6535
2	4.6137	4.6080	4.6023	4.5995	4.5967	4.5910	4.5853
3	4.5455	4.5398	4.5341	4.5313	4.5285	4.5228	4.5171
4	4.4773	4.4716	4.4659	4.4631	4.4603	4.4546	4.4489
5	4.4091	4.4034	4.3977	4.3949	4.3921	4.3864	4.3807
6	4.3409	4.3352	4.3295	4.3267	4.3239	4.3182	4.3125

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Ratio to trend ($y/\hat{y} \times 100$)

	Mon	Tue	Wed	Thu	Fri	Sat
	53.3971	91.9550	94.2083	98.6088	109.4609	133.223
	39.0142	73.7847	112.9870	106.5982	115.4433	148.301
	26.3997	55.0685	105.8646	112.6201	137.0832	154.966
	42.4313	49.1994	109.4203	136.7621	159.3858	155.09
	47.6288	72.6711	97.7784	120.6712	139.0662	164.35
	27.6440	71.5077	62.3629	111.0109	122.7363	143.76
	39.4200	69.0311	97.1536	114.3785	130.5293	149.9
	39.3895	68.9775	97.0782	114.2898	130.4280	149.8
Daily avg ($\bar{x}_i$)	39.4200	69.0311	97.1536	114.3785	130.5293	149.9
Daily seasonal index (S.I) ($\bar{x}_i/\bar{\bar{x}} \times 100$)	39.3895	68.9775	97.0782	114.2898	130.4280	149.8

$\bar{\bar{x}}$ = overall avg.

$= 100.0775$

$\Sigma SI = 600.$

Ex: In a study of sales, a company obtained the following least square trend equation

$y = 16 + 2x$

origin : 1975 , x units = 1 year

y – total no. of units sold per year.

The company has physical capacity to produce only 30 units a year and it believes that atleast for the next decade the trend will continue as before.

(i) What is the average annual increase in the no. of units sold?

(ii) By what year the company's expected sales have equalled to its present capacity?

(iii) Estimate the annual sales for the year 1988.

Ans: (i) The avg. annual increase in the no. of units sold = b = 2 units/year.

(ii) The company has physical capacity to produce only 30 units a year

$\therefore 30 = 16 + 2x \quad \therefore 2x = 14 \quad \therefore x = 7$

i.e. In 7 years the company's expected sales have equalled to its present capacity.

(iii) $y = 16 + 2x$ $\quad$ y : sales (per year) $\quad$ x : year